Competitive ratio of load balancing in the random order arrival model

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In the early days of research on online algorithms people measured the performance of an algorithm by comparing its performance with the optimal solution over the worst case instance. This pessimistic analysis was motivated by the fact that often we don't know an assumption we could make about the instances, so we have to assume the worst case. This lead to the study of the competitive ratio of various problems, with results that seemed to pessimistic in real life.

Recently research focuses on adversarial models which are more realistic. One of them is the random order model. The adversary chooses a worst case set of requests, which then however are presented in uniform random order to the algorithm.

This model has been studied for the following central problem in scheduling. We are given m parallel unrelated machines. Every job j has an adversarial chosen size vector $(p_{ij})_{\text{machines } i}$ and jobs are presented in uniform random order to the algorithm. Each job j has to be assigned to a machine i, where it adds p_{ij} to the load of the machine. The goal is to minimize to maximum load, also called *makespan* in scheduling terms.

Recently it has been shown that the competitive ratio is $O(\log m/\log\log m)$ [Mol17]. This improved over the classical list scheduling algorithm (placing every job on the least loaded machine) which is only $\lceil \log m \rceil + 1$ competitive [ANR95]. Only a simple lower bound of $\Omega(\log\log m)$ is known which is based on concatenating all possible arrival orders into one instance given to the algorithm [Pla17].

The short term goal of this internship is to get familiar with lower and upper bound techniques for load balancing, and to close the gap, probably by constructing a better lower bound. The long term goal is the understanding of the random order model for online optimization problems.

References

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